**Homework on Binomial Trees**

1. **A stock index currently stands at 300. It is expected to increase or decrease by 10% over each of the next two time periods of three months. The risk free interest rate is 8% and dividend yield on the index is 3%. What is the value of a six month put option on the index with a strike price of 300 if it is (a) European and (b) American**

363 (0)

330(0 or 1.29)

300 297 (3)

270 (30 or 26.23)

243 (57)

p=(e^((0.08-0.03)\*0.25)-0.9)/(1.1-0.9)=0.56

t=1:

UP: exp(-0.08\*0.25)\*(0.56\*0+(1-0.56)\*3)=1.29

DOWN: exp(-0.08\*0.25)\*(0.56\*3+(1-0.56)\*57)=26.23

t=0

European put = exp(-0.08\*0.25)\*(0.56\*1.29+(1-0.56)\*26.23)=12.02

American put = exp(-0.08\*0.25)\* (0.56\*1.29+(1-0.56)\*30)=13.65

1. **A futures price is currently 60. It is known that over each of the next two three month periods it will either rise by 10% or fall by 10%. The risk free interest rate is 8% per annum. What is the value of a six month European call option on the futures with a strike price of 60? If the call were American would it ever be worth exercising it early?**

72.6 (12.6)

66(6 or 6.17)

60 59.4 (0)

54(0)

48.6 (0)

u=1.1

d=0.9

∆t=0.25

r=0.08

K=60

S=60

p= (1-0.9)/(1.1-0.9)=0.5

European call = (0.5^2\*12.6+2\*0.5\*(1-0.5)\*0+(1-0.5)^2\*0)\*exp(-2\*0.08\*0.25)=3.0265

American call = 3.0265

1. **A stock price is currently $80. It is known that at the end of four months it will be either $75 or $88. The risk free rate is 6 percent per annum with continuous compounding. What is the value of a four–month European put option with a strike price of $80? Use no-arbitrage arguments. What is the delta of this option and what does delta represent in this setting?**

At the end of four months the value of the option will be either $5 (if the stock price is $75) or 0 (if the stock price is 88)

1. Hedging portfolio = Delta shares of stock +write 1 put

75\*Delta-5=88\*D 🡪D=-0.38

80\*Delta-p=88\*Delta\*Exp(-0.06\*4/12)🡪p=2.41

1. Delta=DP/DS= (5-0)/(75-88)=-0.38: 0.38 shares of stock to be sold for each put written.
2. **A futures price is currently 25, its volatility is 30% per annum, and the risk free interest rate is 10% per annum. What is the value of a nine month European call on the futures with a strike price of 26?**

F0=25, X=26, sigma=0.3, r=0.1, T=0.75

d1=(ln(F0/X)+sigma^2 \*T/2)/(sigma\*sqrt(T))=-0.0211

d2=(ln(F0/X)-sigma^2\*T/2)/(sigma\*sqrt(T))=-0.2809

c=e^(-0.075)(26N(-0.0211)-26N(-0.2809))=2.01